Feb. 2017 Professor S. Boyd

Homework 1

1. *Optimal disposition of a stock.* You must sell a total amount $B > 0$ of a stock in two rounds. In each round you can sell any nonnegative amount of the stock; by the second round all of the initial stock amount B must be sold. The (positive) prices in the two rounds are p_0 and p_1, respectively. These are independent log-normal variables:

$$\log p_0 \sim \mathcal{N}(\mu_0, \sigma_0^2), \quad \log p_1 \sim \mathcal{N}(\mu_1, \sigma_1^2).$$

The goal is to maximize the total expected revenue from the sales in the two rounds.

We consider three different information patterns.

- *Prescient.* You know p_0 and p_1 before you decide the amounts to sell in each period.
- *No knowledge.* You do not know the prices.
- *Partial knowledge.* You are told the price p_0 before you decide how much to sell in period 0, and you are told the price p_1 before you decide how much to sell in period 1.

(a) Formulate this as a stochastic control problem. Identify the state and action spaces, the dynamics map, the stage utility function (*i.e.*, function of state and action that is to be maximized), and horizon length T. You can include a terminal state constraint, *i.e.*, the requirement that x_T must have a certain value.

(b) Derive the optimal policies for each of the three different information patterns. The amount sold in each period can depend on the problem data $(B, \mu_0, \mu_1, \sigma_0, \sigma_1)$ and of course the additional information available, which depends on the information pattern.

(c) *Numerical example.* Consider the specific case with

$$B = 10, \quad \mu_0 = 0, \quad \mu_1 = 0.1, \quad \sigma_0 = \sigma_1 = 0.4.$$

Plot the distribution of total revenue for the stochastic control problems for the three different information patterns, using Monte Carlo. Give the expected values of total revenue in each case (again, computed by Monte Carlo).

Hint.

- If $\log x \sim \mathcal{N}(\mu, \sigma^2)$, we have $\mathbf{E}\, x = \exp(\mu + \sigma^2/2)$.

Remark. No, you don't need to know how to solve stochastic control problems to solve this problem. You can solve it directly using basic and simple arguments.